Math 131B-2: Homework 8

Due: May 30, 2014

1. Read Apostol Sections 1.21-27, 1.29, 1.32, Tao Sections 16.1-2. [You may also wish to compare Tao Chapter 15 to the Apostol reading.]

2. Do Apostol problems 1.27, 1.30, 1.36.

3. Prove the inequalities for the complex norm from class, namely that

   - $|R(z)| \leq |z|$
   - $|I(z)| \leq |z|$
   - $|z| \leq |R(z)| + |I(z)|$
   - $|z + w| \leq |z| + |w|$

4. Prove that $\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ and $\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$.

5. Find the Taylor series expansions for $e^x \sin x$ and $\ln(1 + x)\tan^{-1}(x)$ about 0. Note that if your solution involves the product rule, you're working too hard.

6. Let $f : \mathbb{R} \to \mathbb{R}$ be a function which is differentiable at x_0, with $f(x_0) = 0$ and $f'(x_0) \neq 0$. Show there exists a $c > 0$ such that $f(y) \neq 0$ for $0 < |x - y| < c$. In particular, show there is a $c > 0$ such that $\sin(x) \neq 0$ for $0 < x < c$.

7. Let $\tan(x) = \frac{\sin x}{\cos x}$.

   - Show that the tangent function is differentiable and monotone increasing on $(\frac{-\pi}{2}, \frac{\pi}{2})$, and thus invertible.
   - Show that the derivative of $g(x) = \tan^{-1}(x)$ is $\frac{1}{1+x^2}$. [Hint: What do you know about the derivatives of inverse functions from last quarter?]

8. Prove that a continuous 1-periodic function $f\colon \mathbb{R} \to \mathbb{C}$ is bounded. Give an example showing that if f is not assumed to be continuous, this need not be true.